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SPATIAL COHERENCE AND INTENSITY PROPERTIES OF QUASIHOMOGENEOUS OPTICAL SOURCES.

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- (U) Until 1960 there existed only incoherent sources in the form of arc lamps and discharge lamps. The most striking characteristic of these optical sources is that in addition to being incoherent, they radiate omnidirectionally. In 1960 a new type of light source, now known as the laser, appeared. Remarkably, this new optical radiation source had properties which are distinctly opposite to those of incohrent sources. These properties are intense brightness, monochromaticity, total coherence and very narrow directionality or unidirectionality of the laser radiation. Thus, there now existed two different optical sources with distinctly opposite properties.
- (U) One might have thought that the question would have been asked, "What is the nature of the optical sources and their corresponding fields which are between these two extremes?" Only during the past several years has a new type of optical source whose properties are between these extremes been characterized. This new type of optical source has been named the "quasihomogeneous optical source":
- (U) As mentioned above, one of the most striking properties of the laser is its directionality. From the beginning, the directional property of the laser was associated with its being completely coherent. In fact, this directionality was believed to be due totally to its coherency. The logic here was that incoherent radiation led to omnidirectional radiation, while completely coherent radiation led to unidirectional radiation. This logic implied that light, which is partially coherent, will generate fields or beams with a greater divergence than the laser but less than incoherent sources. This is represented in Fig. 1.

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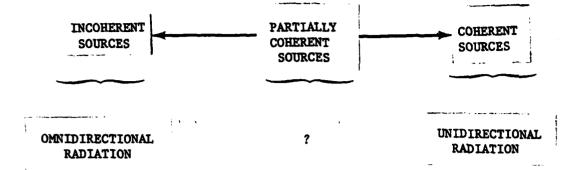


Fig.1. Coherence properties of optical sources. The question mark under the partially coherent sources represents the unknown nature of these sources.

- (U) Recently, theoretical and experimental investigations have shown that partially coherent sources can be as directional as a laser beam. This is due to the fact than an optical source must be simultaneously characterized by its degree of coherence and its intensity distribution across the optical surface. This theoretical prediction has been confirmed experimentally.
- (U) In this paper the theoretical development of the quasihomogeneous optical source concept is reviewed along with some of the experimental results. No attempt will be made to present a detailed derivation of the fundamental equations as they have been fully developed in the open literature and can be found in the cited references.

# Mathematical Formulation of Optical Coherence

 $\left( \text{U} \right)$  The optical field in free space can be characterized by the scalar wave equation

$$\nabla^{2} V(r,t) = \frac{c^{2}}{1} \frac{\partial t^{2}}{\partial t^{2}} (r,t)$$
 (1)

where V(r,t) is the instantaneous field amplitude, c is the speed of light in a vacuum and  $\nabla^2$  is the three dimensional Laplacian operator.

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In accordance with Wolf's formulation, it is possible to introduce another quantity, known as the mutual coherence function, defined as

$$\Gamma(\underline{r}, \underline{r}, \tau) = \langle V(\underline{r}, t+\tau) V^*(\underline{r}, t) \rangle$$
 (2)

where the angle brackets represent a time average of the optical field,  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are two different points in the optical field in space and  $\mathbf{r}_2$ . Ts an increment of time. Wolf has shown that the mutual coherence function, Eq.(2) also satisfies the wave equation. Thus, the coherence properties of the optical field propagate along with the amplitude. It is possible to take the Fourier transform of the mutual coherence function  $\Gamma\left(\mathbf{r}_1,\mathbf{r}_2;\tau\right)$  to form

$$W(r_1, r_2; \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(r_1, r_2; \tau) e^{i\omega\tau} d\tau$$
 (3)

The quantity  $W(r_1, r_2; \omega)$  is known as the cross-spectral density and characterizes the correlations of the optical field at the frequency,  $\omega$ , at two points in space,  $P(r_1)$  and  $P(r_2)$ . The mutual degree of coherence,  $\psi(r_1, r_2; \omega)$ , can be defined in terms of the cross-spectral density  $W(r_1, r_2; \omega)$ 

$$\mathcal{M}(\Gamma_{1},\Gamma_{2};\omega) = \frac{\mathcal{W}(\Gamma_{1},\Gamma_{2};\omega)}{\sqrt{\mathcal{I}(\Gamma_{0},\omega)\mathcal{I}(\Gamma_{2},\omega)}}$$
(4)

where I( $\underline{r}$ ,  $\omega$ ) = W( $\underline{r}$ , $\underline{r}$ ;  $\omega$ ) represents the average optical intensity at frequency  $\omega$ , at the point P( $\underline{r}$ ). It can be shown that  $\mu$  ( $\underline{r}_1$ , $\underline{r}_2$ ; $\omega$ ) is normalized so that for all values of  $\underline{r}_1$ ,  $\underline{r}_2$  and  $\omega$ ,

The quantity  $\mu(\underline{r}_1,\underline{r}_2;\omega)$ , defined by Eq.(4), is called the complex degree of spatial coherence of the light fluctuations at frequency  $\omega$  at the points  $P(\underline{r}_1)$  and  $P(\underline{r}_2)$ . The limiting values of unity and zero in Eq.(5) indicate that the light fluctuations at the points  $P(\underline{r}_1)$  and  $P(\underline{r}_2)$  are completely correlated or uncorrelated, respectively. If  $\{\mu(\underline{r}_1,\underline{r}_2;\omega)\} = 1$  then the optical field is said to be spatially coherent. On the other hand if the value of  $\{\mu(\underline{r}_1,\underline{r}_2;\omega)\} = 0$ 

then the optical field is said to be completely spatially incoherent. These limiting cases should be regarded only as convenient mathematical idealizations rather than real physical conditions actually observed in nature. No practical optical field can be spatially incoherent in the sense defined above.

(U) By suppressing the time factor in Eq.(2) one can show that the cross-spectral density function,  $W(\underline{r}_1,\underline{r}_2)$ , obeys the Helmholtz equation

$$\Delta_{s}^{i} M(\overline{L}^{i}, \overline{L}^{r}) + K_{s} M(\overline{L}^{i}, \overline{L}^{r}) = 0$$
 (9)

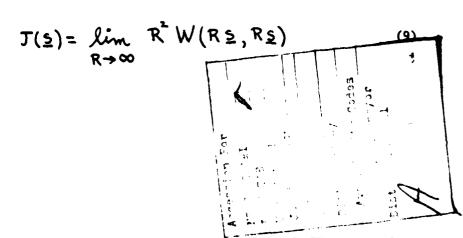
where the index i on the Laplacian operator indicates differentiation with respect to either variable  $\underline{r}_1$  or  $\underline{r}_2$ . By using standard mathematical techniques for solving the Helmholtz equation, the cross-spectral density function in the optical far-field can be related to its values at all pairs of points in the source plane. More specifically, the far-field solution of Eq.(6) takes the form

$$W(\underline{r}_1,\underline{r}_2) = \left(\frac{2\pi K}{R}\right)^2 \cos^2 \theta \ \widetilde{W}^{(0)}(K\underline{S}_1,-K\underline{S}_1) \tag{7}$$

where W  $^{(0)}(\underline{k}_1$  , $\underline{k}_2$  ) is the spatial Fourier transform of the cross-spectral density in the source plane

$$\underset{(0)}{\underbrace{M}}_{(0)}(\overline{K}^{1},\overline{K}^{5}) = \frac{(3\pi)_{H}}{1} \iiint_{\infty} \underset{(0)}{\underbrace{M}}_{(0)}(L^{1},\overline{L}^{5}) \underset{-\Gamma(\overline{K}^{1},L^{1}+\overline{K}^{5},L^{2})}{\underbrace{q_{5}L^{1}q_{5}L^{5}}}$$
(8)

The vector s is the projection of the unit vector s in the plane of the source and s is the angle between s and the normal to the source plane. As has been pointed out above the cross-spectral density is useful for defining the mutual degree of coherence, Eq.(4). In addition, one can also show that the radiant intensity of physical optics, J(s), is directly proportional to the diagonal element of the cross-spectral density, namely,



where s is the unit vector in the direction of observation. With this definition, Eq. (7) becomes

$$J(\underline{s}) = (2\pi K)^2 \cos^2 \theta \ \widetilde{W}^{(0)}(K \underline{s}_L, -K \underline{s}_L)$$
 (10)

In order to analytically determine  $J(\underline{s})$  the Fourier transform of the cross-spectral density on the optical surface,  $W^{(0)}(k\underline{s}_1,-k\underline{s}_2)$  must be known. In order to do this we substitute Eq.(4) into Eq.(8) and we find

$$\widetilde{W}_{(Q)}(\overline{K}',\overline{K}^{s}) = \frac{(SL)_{H}}{1} \iiint_{\infty}^{-\infty} \left\{ q_{x}\overline{L}' q_{x}\overline{L}' = \frac{(K',L'+K^{2},L^{2})}{m(L',L^{2})} \right\}$$
(11)

Unfortunately, Eq.(11) cannot be evaluated unless the term  $\mu$  (r1, r2) x  $\sqrt{I(r1)}$  · I(r2) can be simplified in a manner which conforms to physical reality. For many years the inability to reduce this factor prevented further progress in the development of partially coherent sources.

(U) This impasse was overcome by Wolf and Carter in the following way.  $^{\rm l}$  The expression for the cross-spectral density

$$W(\underline{\Gamma}_{1},\underline{\Gamma}_{2}) = \mu(\underline{\Gamma}_{1},\underline{\Gamma}_{2})\sqrt{\underline{I}(\underline{\Gamma}_{1})\cdot\underline{I}(\underline{\Gamma}_{2})}$$
(12)

can be cast into a form which agrees with physical reality by noting that most optical sources are statistically homogeneous. This behavior can be expressed by writing

$$\mathcal{M}(\underline{\Gamma}_1,\underline{\Gamma}_2) = g(\underline{\Gamma}_1 - \underline{\Gamma}_2) \tag{13}$$

The treatment of the next factor,  $\sqrt{I(r_1)} \cdot I(r_2)$  is more subtle and difficult and requires a considerable amount of insight. Investigation of optical sources shows the optical intensity, I(r), changes very slowly with position across the source and is sensibly constant over regions whose linear dimensions are of the order of the correlation distance of the light source. Under these conditions, the intensity varies very slowly across the source and so we can express this behavior by

$$\sqrt{I(r_i) \cdot I(r_k)} \simeq I\left[\frac{1}{2}(r_i + r_k)\right]$$
 (14)

Thus, a source which is statistically homogeneous, Eq. (13), and whose intensity varies slowly across the source, Eq. (14), allows one to write

$$W(\underline{r}_1,\underline{r}_2) \simeq g(\underline{r}_1 - \underline{r}_2) I\left[\frac{1}{2}(\underline{r}_1 + \underline{r}_2)\right]$$
 (15)

An optical source which behaves in accordance with the previous description and satisfies Eq.(15) is said to be quasihomogenous. From Eq.(15) the Fourier transform of the cross-spectral density can be shown to be

$$\widetilde{W}(\overline{K}^{1},\overline{K}^{2}) = \widetilde{\mathbf{I}}(\overline{K}^{1} + \overline{K}^{2}) \widetilde{\mathcal{G}}\left[\frac{1}{2}(\overline{K}^{1} - \overline{K}^{2})\right]$$
(15')

Evaluation of Eq.(11) on the surface of the optical source we see that the total contribution to the optical far-field radiant intensity,  $J(\underline{s})$ , is then

$$\mathcal{J}(\underline{s}) = (\mathcal{Z}\pi K^2) \hat{\mathcal{I}}^{(0)} \hat{\mathcal{G}}^{(0)}(K\underline{s}^2) COS^2\Theta$$
 (16)

Therefore, we have reached the important result that the radiant intensity, J(s), depends both on the intensity distribution over the source, I(0), and the mutual degree of coherence, g(0). From Eq.(16) we that we need only know the Fourier transform of the intensity distribution over optical source and the Fourier transform of the correlation function in order to determine the far-field radiant intensity, J(s).

### Far-Field Radiant Intensity for a Gaussian Intensity and Coherence Source

(U) We will now consider an optical source whose intensity distribution and degree of spatial coherence are both gaussian and which can be represented in the form

$$I(I) = A e \times P(-r^2/2\sigma_I^2)$$
 (17a)

$$g(\underline{r}) = e \times p(-r^2/2\sigma_8^2) \tag{17b}$$

Taking the Fourier transform of Eqs. (17a) and (17b) and substituting these transforms into Eq. (16) we find the radiant intensity in terms of its angular spectrum to be

$$J(\theta) = T(0)\cos^2\theta \exp(-\sin^2\theta/2\Delta^2)$$
 (18)

where  $\theta$  is the polar angle of observation as measured from the z axis, and

$$\Delta^2 = \frac{1}{(K\sigma_a)^2} + \frac{1}{(2K\sigma_a)^2}$$
 (19a)

and

$$T(o) = \left(\sigma_{x}^{2} / \Delta\right) A \tag{19b}$$

The angular half-width of the optical field can be found by setting the argument in exponent of Eq.(18) to -2 (the half power point), and we find that

$$\theta = \sin^{-1}\left[2\sqrt{\frac{1}{(\kappa\sigma_{k})^{2}} + \frac{1}{(2\kappa\sigma_{k})^{2}}}\right]$$
 (20)

From Eq.(20) we see that the angular intensity distribution is a funct-of the mean intensity width,  $\sigma_{\downarrow}$ , and the correlation width,  $\sigma_{\downarrow}$ , across the optical source. Let us now consider some special cases.

Laser

For a laser,  $\sigma_g \longrightarrow \infty$  , and we have

$$\Delta_{L} = \frac{1}{(2\kappa\sigma_{L})} = \frac{\lambda}{4\pi\sigma_{L}} \qquad (21a)$$

and

$$\theta_{L} = \sin^{-1}\left[\frac{\lambda}{2\pi\sigma_{L}}\right] \tag{21b}$$

For a typical HeNe laser,  $\lambda = 6.328 \times 10(-7) \text{m}$  and  $\sigma_{\text{L}} = 5 \times 10(-4) \text{m}$ .

Substituting these values into Eq. (21b) the angular half-width in the far field is

$$\theta_{L} \simeq 11.5 \, \text{mrad}$$
 (22)

This result shows that we may approximate  $\theta_{T}$  by

$$\theta_{L} \simeq \lambda/\sigma_{L}$$
 (23)

In this case,  $\sigma_{I} \longrightarrow \infty$ , and  $\sigma_{g} \longrightarrow 0$  so

$$\Delta_{INC} = \frac{1}{K \sigma_{g}}$$
 (24a)

and

$$\theta_{\rm INC} = 51N^{-1} \left[ \frac{\lambda}{2\pi \sigma_{\rm s}} \right]$$
 (24b)

We note that  $\lambda \sim 10(-7)m$ ,  $\sigma \longrightarrow 0$  and therefore  $\theta \longrightarrow 0$  will become large as  $\sigma \longrightarrow 0$  in agreement with the well known behavior of incoherent sources.

Partially Coherent Sources

Partially coherent sources are defined to be optical sources whose deree of coherence is greater than 0 but less than 1. We now show that it is possible to construct a source which is as directional as a laser but which is partially coherent. To see this we know from Eq. (21b) that

 $\theta_L = \sin^{-1}\left[\frac{\lambda}{2\pi\sigma}\right]$ (25)

Equating Eq. (20) for the laser and the quasihomogenous source, the requirement to have the same beam divergence is

$$\frac{1}{\sigma_{q}^{2}} + \frac{1}{4\sigma_{q}^{2}} = \frac{1}{4\sigma_{L}^{2}}$$
 (26)

Thus, if  $\sigma_{\hat{L}}$  of the laser is taken to be equal to 1/4 we then have

$$\frac{1}{\sigma_{\lambda}^{2}} + \frac{1}{+\sigma_{\alpha}^{2}} = 1 \tag{26'}$$

If, we now choose  $\sigma_{gQ} = \sqrt{2}$  then  $\sigma_Q$  must be equal to  $1/\sqrt{2}$ . This trade-off behavior is shown in Fig.2, below. The top figure is that of a laser while the following figures illustrate the variation in the mean width of the intensity and coherence to obtain the same angular divergence.

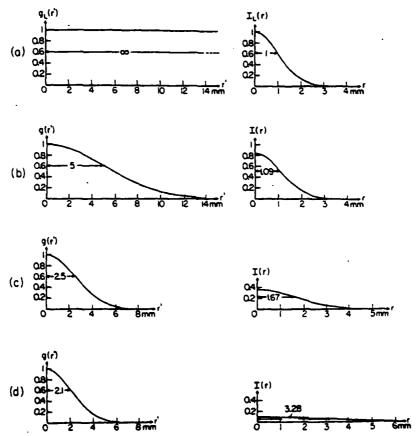


Fig. 2 illustrating the coherence and the intensity distributions across three partially coherent sources [(b), (c), (d)] which produce fields whose far-zone intensity distributions are the same as that generated by a coherent laser source [(a)]. The parameters characterizing the four sources are:

(a)  $\sigma_g = -$ ,  $\sigma_f = \delta_L = 1$  mm, A = 1 (arbitrary units) (b)  $\sigma_g = 5$  mm,  $\sigma_f = 1.09$  mm, A = 0.84

(c)  $\sigma_g = 2.5 \text{ mm}$ ,  $\sigma_f = 1.67 \text{ mm}$ , A = 0.36 (d)  $\sigma_g = 2.1 \text{ mm}$ ,  $\sigma_f = 3.28 \text{ mm}$ , A = 0.09.

The normalized radiant intensity generated by all these sources is  $J(\theta)/J(0) = \cos^2\theta \exp\left\{-2(k\delta_L)^2 \sin^2\theta\right\}$ ,  $(\delta_L = 1 \text{ mm})$ .

### Quasihomogeneous Source Experiments

(U) In order to test the validity of the theoretical conclusions presented in the previous sections, a series of laboratory measurements were made in the far-field of a quasihomogeneous optical source. At this time there are no primary quasihomogeneous sources, that is, sources which radiate directly in the quasihomogenous mode. However, it is possible to construct a secondary source which behaves quasihomogeneously, i.e., is statistically homogeneous and whose intensity varies slowly over the source. In order to obtain the statistical homogeneity a gaussian phase screen was constructed. A laser has a natural gaussian intensity and this was used as a primary source. The quasihomogeneous source was created by expanding and collimating the laser into a 25mm beam diameter. This beam was then passed through the phase plate which was rotating at ten Hertz. The mean width of the intensity of the emerging beam is then 5.82mm. The coherence width of the phase plate was measured indirectly. However, by means of indirect measurements made on two phase plates used in the experiments values of  $\sigma$  = 8.8  $\mu$ m and 85.4  $\mu$ m were found. The experimental setup is shown in Fig.3. The fact should be emphasized that the use of the laser as a primary source was to obtain a natural gaussian intensity distribution; by passing the laser beam through the rotating phase plate the coherence of the laser beam was reduced thereby making a quasihomogeneous beam.

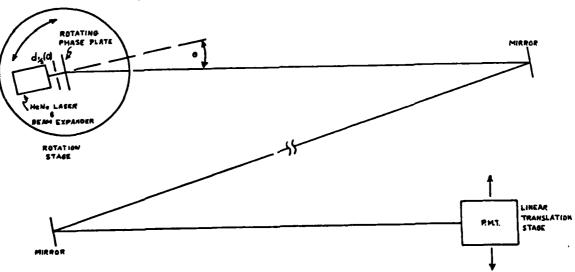


Fig. 3. Experimental arrangement to determine the far-field properties of a quasihomogeneous optical source. The total path length between the rotating phase plate and the PMT was varied by placing additional mirrors in the optical path. With as many as eight mirrors the total path was 12.5m. The rotational stage was made in order to align the quasihomogeneous source with the scanning PMT.

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(U) In terms of carrying out a series of measurements a convenient quantity to measure is the beamwith,  $d_{1}/(z)$ , as a function of the pathlength, z. The expression which shows this relation is given by

$$d_{1/2}(z) = 2\sqrt{\ln 2}\sqrt{\left(2\sigma_{a}^{2} + \frac{22^{2}}{\kappa^{2}\sigma_{a}^{2}}\right)}$$
 (27)

With the values of  $\sigma_{\rm g}$  and  $\sigma_{\rm I}$  given above, the beam width as function of z was measured and the results are shown in Fig. 4. The solid lines represent the theoretical values expressed by Eq.(27). and the dots are the measured values. The agreement is seen to be very good.

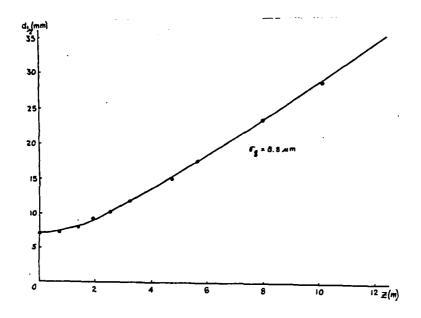


Fig. 4. Experimental confirmation of the quasihomogeneous source farfield behavior. The solid line represents the theoretical value of  $d_{1/2}(z)$  and the solid circles the measured values.

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# Military Applications

- (U) It is worthwhile to devote a short section to the possible role of quasihomogeneous optical sources in military systems.
- (U) At present there are four known distinct properties of light. These are 1) the intensity, 2) the wavelength, 3) the polarization and 4) the coherence. The intensity property of light has been used by the military since antiquity. With the advent of spectrally pure lasers the wavelength property also came into use. As for the polarization property it is only now being applied in a prottype Army system. The coherency property has never been used but this may change. With the development of the current line of research new methods have been developed to control and generate coherence. Thus, new applications to an unused domain of light are now possible.
- (U) To give some examples and possible applications it should now be possible to transmit signals and messages by coherency coding, that is, modulate the laser with a phase plate and then coherently demodulate the received signal. This should find direct use in laser designators and transmission over fiber optics links. Another application is in image improvement by the reduction of laser speckle. By reducing the coherency of the laser beam experiments have shown a dramatic improvement in the image quality. Another application is to the development of laser warning receivers where the variability of coherence adds a new dimension to the detection criteria. Finally, quasihomogeneous sources may find possible application in the manufacture of integrated circuits where the use of this new type of source can reduce and control diffraction effects in the making of photolithographic masks.
- (U) Thus, for the military, the development of quasihomogeneous optical sources and the techniques which are now being developed to generate and control the optical field may open the optical coherency domain as a new area of military exploitation.

### Summary

- (U) In this paper we have introduced the concept of a new type of optical source whose properties are between incoherent optical sources and coherent laser sources. The conditions were established for a source to be quasihomogeneous. In doing this the far-field radiant intensity, J(s), was shown to directly dependent on both the intensity distribution and the degree of coherence across the optical source.
  - (U) This last result led to the following realization. For a

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quasihomogeneous optical source with a gaussian intensity and coherence function it is possible to generate beams as directional as a laser. This can be done even though the degree of coherence is reduced. However, the mean intensity width of the beam must then be increased in a prescribed manner. This, fortunately, is not difficult to do and the expected behavior has been confirmed experimentally.

(U) Finally, several military applications of the use of quasihomogeneous optical sources was presented. As a result of this present line of research it may now be possible to open up a new area of optics for military exploitation.

## References

- 1. W.H. Carter and E. Wolf, "Coherence and radiometry with quasihomogeneous planar sources", J. Opt. Soc. Am. 67, 785-796(1977).
- 2. E. Wolf, "The radiant intensity from planar sources of any state of coherence", J. Opt. Soc. Am 68, 1597-1605(1978).
- 3. E. Collett and E. Wolf, "Is complete spatial coherence necessary for the generation of highly directional light beams?" Opt. Lett. 2, 27-29(1978).
- 4. E. Wolf and E. Collett, "Partially coherent sources which produce the same far-field intensity distribution as a laser", Opt. Commun. 25, 293-296(1978).
- 5. F. Gori and C. Palma, "Partially coherent sources which give rise to highly directional beams", Opt Commun. 27,185-188(1978)
- 6. J.D. Farina, L.M. Narducci and E. Collett, "Generation of highly directional beams from quasihomogeneous optical sources", accepted for publication in Opt. Commun.
- 7. M. Born and E. Wolf, <u>Principles of Optics</u>, 5th ed. (Pergamon, 0x-ford, 1975).